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Proving the Point: Connections Between Legal and Mathematical Reasoning

Maria Termini
Brooklyn Law School, maria.termini@brooklaw.edu

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ABSTRACT

"I think there are a lot of people who go to law school because they are not good at math and can't think of anything else to do."[1] Chief Justice John Roberts got a good laugh at that line when speaking to students at Rice University. The stereotype about lawyers disliking math is widely held, including by lawyers themselves. But the disciplines of law and mathematics are more closely connected than many lawyers suspect. If those lawyers who are "not good at math" took an upper-level college mathematics course—one focusing more on theorems and reasoning than on numbers and calculations—they might find it far easier and more familiar than they expect. This Article explores the connections between mathematical analysis and legal analysis—including their similar organizational schemes, types of reasoning, and purposes—and draws lessons for legal writing from the realm of mathematics.

I. INTRODUCTION

Fermat's Last Theorem was a famous unsolved problem for three and a half centuries, and its solution made headlines even outside of mathematical circles. The Theorem takes its name from Pierre de Fermat, an avid amateur mathematician who died in 1665.[2] After Fermat's death, among his possessions was a mathematics text that included his notes related to the Pythagorean Theorem. The Pythagorean Theorem gives a formula for determining the lengths of the sides of a right triangle. If c is the hypotenuse of a right triangle—i.e., the side opposite the right angle—and a and b are the other two sides of the right triangle, then $a^2 + b^2 = c^2$.[3] While the lengths of the sides of a right triangle are

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* Assistant Professor of Legal Writing, Brooklyn Law School. I am grateful to David J. Ziff, K. Sabeel Rahman, Jocelyn Simonson, Jayne Ressler, Heidi K. Brown, Meg Holzer, Natalie Chin, Kate Mogelescu, Carrie Teitcher, Joy Kanwar, and Rebecca Rogers for their helpful comments, to Kathleen Darvil, Sara-Catherine Gerdes, and Lauren Cedeno for their research assistance, and to the Brooklyn Law School Dean's Summer Research Stipend Program for financial support.

1. Rice Univ., *Centennial Lecture Series: Chief Justice John Roberts Speaks at Rice University*, YouTube (Oct. 26, 2012), https://www.youtube.com/watch?v=UxaFhJ8JVq8 [https://perma.cc/4CAJ-UXQJ].

2. SIMON SINGH, FERMAT'S ENIGMA: THE EPIC QUEST TO SOLVE THE WORLD'S GREATEST MATHEMATICAL PROBLEM 35, 62 (1997).

3. *See id.* at 18-19 (explaining Pythagorean Theorem).

not always whole numbers, it is not difficult to find whole numbers for *a*, *b*, and *c* that fit the formula. For example, *a*, *b*, and *c* could be 3, 4, and 5, respectively, because $3^2 + 4^2 = 5^2$. Another example is 5, 12, and 13. These are known as Pythagorean triples.[4]

The Pythagorean Theorem and Pythagorean triples appear in *Arithmetica*, a book written in approximately 250 C.E. by Diophantus, a scholar in Alexandria.[5] Centuries later, while studying his Latin copy of *Arithmetica*, Fermat considered the Pythagorean Theorem and variations of Pythagorean triples.[6] Specifically, he wondered whether any solutions could be found for *a*, *b*, and *c* when they are raised to any power greater than two.[7] Are there any whole number solutions when *a*, *b*, and *c* are each cubed? When they are each raised to the fourth power or the fifth power? Fermat thought the answer to each question was "no."[8] In the margin of his copy of *Arithmetica*, Fermat stated that for any whole number *n* greater than two, there are no whole numbers *a*, *b*, and *c* that make the equation $a^n + b^n = c^n$ true.[9] Then, in Latin, Fermat noted: "I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain."[10] After Fermat's death, his son found and published this and other mathematical notes, but Fermat's proof of his "last theorem" was never found.[11] For decades, and then centuries, other mathematicians tried to recreate or discover the proof for themselves, without success.[12]

In 1986, Andrew Wiles, then a professor of mathematics at Princeton University, set himself the task of proving Fermat's Last Theorem.[13] He worked on the project for seven years, mostly alone and in secret.[14] In 1993, Wiles

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4. *See id.* at 28 (defining and providing examples of Pythagorean triples).

5. *See id.* at 51, 60 (describing life and work of Diophantus of Alexandria).

6. *See* SINGH, *supra* note 2, at 60-62 (describing formulas Fermat considered).

7. *See id.* at 61 (outlining Fermat's thought process).

8. *See id.* (quoting and translating Fermat's assertion no such whole number solutions exist when power exceeds two).

9. *See id.* (stating Fermat's theory).

10. *See* SINGH, *supra* note 2, at 62 (quoting and translating Fermat's marginal notation).

11. *See id.* at 62-63 (describing search for proof of theorem).

12. *See* James Gleick, *Fermat's Theorem Solved? Not This Time*, N.Y. TIMES (Mar. 29, 1988), https://www.nytimes.com/1988/03/29/science/fermat-s-theorem-solved-not-this-time.html [https://perma.cc/GU5Q-UT6T] (noting "[a]mateurs and professionals alike . . . struggled for 350 years" attempting to prove Fermat's Last Theorem).

13. *See* SINGH, *supra* note 2, at 204-05 (noting Wiles inspired by another mathematician's proof linking Fermat's Last Theorem to Taniyama-Shimura conjecture).

14. *See* Gina Kolata, *How a Gap in the Fermat Proof Was Bridged*, N.Y. TIMES (Jan. 31, 1995), https://www.nytimes.com/1995/01/31/science/how-a-gap-in-the-fermat-proof-was-bridged.html [https://perma.cc/57U7-W95T]. The story of Wiles working alone fits the popular image of a mathematician in solitude, but that image is largely a myth. *See generally* REUBEN HERSH & VERA JOHN-STEINER, LOVING AND HATING MATHEMATICS: CHALLENGING THE MYTHS OF MATHEMATICAL LIFE chs. 5, 6 (2011) (refuting myth of mathematicians working in isolation). A mathematician at the opposite end of the teamwork spectrum was Paul Erdös, who was known to be a prolific collaborator. *See id.* at 191. Much like six degrees of separation, mathematicians refer to their Erdös numbers to describe how closely connected they are to Erdös through

announced a solution in a series of three lectures at Cambridge University, but the proof had to be reviewed by referees before publication.[15] As with everything related to Fermat's Last Theorem, the peer review process was not easy. During the review process, one of the referees discovered a problem with the proof.[16] Wiles went back to work to try to address the gap the referee had identified.[17] With another year of work, and in collaboration with his former student, Richard Taylor, Wiles was able to bridge the gap, though he nearly gave up in the process.[18] Ultimately, the proof was published as two separate but related papers, which together totaled 130 pages and filled the May 1995 volume of the Annals of Mathematics.[19]

Andrew Wiles is a professional mathematician. His entire career has consisted of studying and teaching mathematics in academia. For Pierre de Fermat, on the other hand, mathematics was a hobby, though a very serious one; Fermat's career was in the law.[20]

This connection between law and mathematics in Fermat's life is not as surprising as it might seem at first blush. Legal reasoning and mathematical reasoning are profoundly connected. Abraham Lincoln explained that studying mathematics helped him better prepare for his legal studies:

> In the course of my law-reading I constantly came upon the word demonstrate. I thought, at first, that I understood its meaning, but soon became satisfied that I did not. . . . At last I said, 'L[incoln], you can never make a lawyer if you do not understand what demonstrate means;' and I left my situation in Springfield, went home to my father's house, and staid there till I could give any propositions in the six books of Euclid[21] at sight. I then found out what 'demonstrate' means, and went back to my law studies.[22]

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scholarly collaborations. *Id.* at 155. Someone who collaborated with Erdös has an Erdös number of 1, someone who collaborated with one of Erdös's collaborators has an Erdös number of 2, and so on. *Id.*

15. *See* SINGH, *supra* note 2, at 244, 253 (describing announcement and publication process).

16. *See id.* at 257 (relating referee's communications with Wiles about error).

17. *See id.* at 259 (describing Wiles's initial attempts to fix error).

18. *See* Kolata, *supra* note 14 (detailing Wiles's successful efforts to mend his proof).

19. *See* SINGH, *supra* note 2, at 276-79 (explaining publication structure of Wiles's proofs). *See generally* Richard Taylor & Andrew Wiles, *Ring-Theoretic Properties of Certain Hecke Algebras*, 141 ANNALS MATHEMATICS 553 (1995); Andrew John Wiles, *Modular Elliptic Curves and Fermat's Last Theorem*, 141 ANNALS MATHEMATICS 443 (1995).

20. SINGH, *supra* note 2, at 35-36 (outlining Fermat's career).

21. Euclid was the Greek mathematician and author who systematically explained mathematics in the several books of his work, *Elements*. *See* Jeremy Gray, *Geometry*, *in* THE PRINCETON COMPANION TO MATHEMATICS 83, 83-84 (Timothy Gowers et al. eds., 2010). Euclid begins *Elements* with some basic definitions and postulates, which are statements that seem to be self-evident but cannot be proven. *See id.* at 84; *see also* H.S.M. COXETER, NON-EUCLIDEAN GEOMETRY 1-2 (6th ed. 1998) (explicating nature of postulates). Euclid then explains geometry by starting with those definitions and postulates and using them to prove theorems. *See* Gray, *supra*, at 84. Thus, Euclid's *Elements* consists largely of mathematical proofs. *Id.*

22. *See Mr. Lincoln's Early Life.; How He Educated Himself*, N.Y. TIMES (Sept. 4, 1864), https://www.nytimes.com/1864/09/04/archives/mr-lincolns-early-life-how-he-educated-himself.html [https://perma.cc/C9WK-JJ

Lincoln's idea—to study mathematical reasoning in order to become better at legal reasoning—might seem strange, especially to those lawyers who do not like math. But law and mathematics have much in common, starting with the basic organizational structure of written analysis in each field. The logical structure of written proofs in upper-level mathematics bears a close resemblance to the basic organizational structure in legal writing: Issue, Rule, Application, Conclusion—often referred to by its acronym IRAC. With a proof, as in IRAC, a writer starts by telling the reader where the writer wants to go (I); then lists the rules or theorems already known (R); applies the known rules to the facts or "givens" (A); and reaches a conclusion based on that reasoning (C).[23]

In addition to using similar organization, both legal and mathematical analysis use the same types of reasoning, including deductive reasoning, inductive reasoning, and arguments in the alternative. Written legal analysis also has similar purposes to those of written mathematical analysis. Both forms of writing aid the author's thought process and are used to convince the reader that the author is correct, to extend the body of knowledge within the field, and to teach those new to the field.

While a comparison between law and mathematics may seem inapt because law is less determinate and less certain than mathematics, a closer look at mathematics reveals this lack of certainty is yet another point of similarity between these two fields.[24] Mathematics, far from revealing some universal truths, can only establish truths based on the assumptions made within the mathematical system being used. Similarly, lawyers cannot find "the law" by looking for universal truths, but instead must consider the laws as they exist within a given legal system.[25] Furthermore, just as existing enacted law and common law cannot address every possible scenario that may come before a court, it is impossible to develop a robust mathematical system in which all mathematical statements can be proven to be true or false.

This Article proceeds as follows. Part II explores the similarities between mathematical analysis and legal analysis, including their similarities of organization, purpose, and types of reasoning. Part III addresses potential differences between law and mathematics, concluding that those differences are not as great as they appear. Part IV considers explanations from cognitive science about the effectiveness of these forms of written analysis. Part V draws lessons for legal analysis in light of the comparison to mathematical analysis.

GC] (recounting J.P. Gulliver's conversation with Abraham Lincoln).

23. *See infra* Section II.A.2 (describing IRAC organization structure and several variations on basic IRAC format).

24. *See infra* Part III (evaluating similarities between law and mathematics).

25. *See generally* H. L. A. Hart, *Positivism and the Separation of Law and Morals*, 71 HARV. L. REV. 593 (1958) (discussing ideals of legal positivism).

II. HOW IS LEGAL ANALYSIS LIKE MATHEMATICAL ANALYSIS?

Written legal analysis and mathematical proof are similar in several ways. In addition to organizational parallels, they also use similar reasoning and share many comparable purposes.

A. Organization

1. Mathematics

Most high school students in the United States learn the "two-column proof" in geometry class. Two-column proofs provide a structure in which students can work through and explain their reasoning when proving a theorem. A two-column proof typically begins with the "givens"—the facts that should be assumed—and a statement of what will be proven. After that, the proof is laid out in two columns. The left-hand column contains numbered statements, which show the proof writer's reasoning, starting with the given facts and building on those facts one step at a time by applying rules to the facts. For each statement in the left-hand column, the right-hand column contains the "rule" that was applied to reach the statement. Each rule must be a given, a definition, a property,[26] a postulate, an axiom,[27] or a previously proven theorem.[28] The proof proceeds step-by-step, ending when the fact to be proven is the final statement in the left-hand column and it is justified by a valid reason in the right-hand column. Figure 1, below, shows an example of a simple two-column proof.[29]

26. The properties of equality are basic statements that are always true of equations. For example, the most basic property, the Reflexive Property of Equality, states that, for any number x, $x = x$. In other words, any number is equal to itself. *See generally Properties of Equalities*, MATH PLANET, https://www.mathplanet.com/education/algebra-1/how-to-solve-linear-equations/properties-of-equalities [https://perma.cc/53AM-GE7D].

27. Postulates and axioms are statements that are assumed to be true without proof. For example, one postulate in Euclidean geometry states that a straight line may be drawn through any two points. *See* COXETER, *supra* note 21, at 1 (identifying Euclid's postulates).

28. *See* RICHARD HAMMACK, BOOK OF PROOF 87 (2d ed. 2013) (discussing key terms in proofs). "A theorem is a mathematical statement that is true and can be (and has been) verified as true. A proof of a theorem is a written verification that shows that the theorem is definitely and unequivocally true." *Id.* The difference, then, between postulates and theorems is that theorems can be proven true, while postulates are accepted as true but cannot be proven. *See supra* note 27 and accompanying text (explaining postulates).

29. The reasons below are presented in a way typical of a high school geometry class, using definitions and properties that would be familiar to students taking the class. For readers who may have imperfect memories of their high school days, this Article includes additional explanations in footnotes.

Figure 1: Two-Column Proof

Given: Angle *A* and Angle *B* are supplementary angles. Angle *B* and Angle *C* are supplementary angles.

Prove: The measure of Angle *A* = the measure of Angle *C*.[30]

Statements	Reasons
1. Angle *A* and Angle *B* are supplementary angles.	Given
2. Angle *B* and Angle *C* are supplementary angles.	Given
3. The measure of Angle *A* + the measure of Angle *B* = 180 degrees.	Supplementary Angles Axiom[31]
4. The measure of Angle *C* + the measure of Angle *B* = 180 degrees.	Supplementary Angles Axiom
5. The measure of Angle *A* + the measure of Angle *B* = the measure of Angle *C* + the measure of Angle *B*.	Transitive Property of Equality[32]
6. The measure of Angle *A* = the measure of Angle *C*.	Subtraction Property of Equality[33]

After geometry, many students never see mathematical proofs again. The proof, however, is an important part of advanced mathematics. In upper level mathematics courses—and with mathematicians generally—written proofs in paragraph form are preferred over two-column proofs.

In many respects, these "paragraph" proofs are like two-column proofs. They typically begin with the givens and with a statement of what is to be proven, and move on to a series of statements and the justifications for those statements, building logically to the conclusion.[34] The main distinction is the form of the

30. This Article uses basic mathematical symbols such as the equal sign, the plus sign, and the like. However, in the interest of clarity for a non-mathematical audience, this Article omits other standard mathematical symbols and instead uses words and phrases to convey the same meaning. For example, this Article uses the phrase "the measure of Angle A" rather than the symbolic representation of that phrase.

31. *See* WALTER J. MEYER, GEOMETRY AND ITS APPLICATIONS 39 (2d ed. 2006) (stating Supplementary Angles Axiom). "If two angles are supplementary, then their measures add to 180[] [degrees]." *Id.*

32. The Transitive Property of Equality states that, for any numbers x, y, and z, if $x = y$ and $y = z$, then $x = z$. In other words, if two numbers are equal to the same third number, then they are also equal to each other. Here, since steps three and four each contain an expression that is equal to 180 degrees, those two expressions are also equal to each other.

33. The Subtraction Property of Equality states that, for any numbers x, y, and z, if $x = y$, then $x - z = y - z$. In other words, if two numbers are equal and you subtract the same thing from each number, the results will also be equal. Here, the measure of Angle B is subtracted from each side of the equation.

34. *See* EUGENIA CHENG, UNIV. OF CHI., DEP'T OF MATHEMATICS, HOW TO WRITE PROOFS: A QUICK GUIDE 3 (2004), http://cheng.staff.shef.ac.uk/proofguide/proofguide.pdf [https://perma.cc/6JYU-4B26] (explaining steps to writing proofs). "A proof is a series of statements, each of which follows logically from what has gone before. It starts with things we are assuming to be true. It ends with the thing we are trying to

statements and justifications of each step of the proof. Instead of listing those steps rigidly in a two-column format, mathematicians usually explain themselves using complete sentences and paragraphs.[35] Figure 2, below, shows an example of a "paragraph" proof.

Figure 2: Proof

Theorem: If x and y are even integers,[36] then the sum of x and y is even.[37]

Proof. Suppose x and y are even integers.[38] Since x is an even integer, there must be an integer n such that $x = 2n$.[39] Similarly, since y is also even, there must be an integer m such that $y = 2m$. Thus, $x + y = 2n + 2m = 2(n + m)$. Since n and m are both integers, $n + m$ must also be an integer. Then, since $x + y = 2(n + m)$ and $n + m$ is an integer, $x + y$ is even.[40]

2. *Law*

This basic proof structure—starting with the thing to be proven, stating rules and applying them to the facts, and then reaching a conclusion—bears a remarkable resemblance to the IRAC organizational scheme used in legal writing.[41] Just as most geometry students will learn about two-column proofs, most beginning law students will learn about IRAC. IRAC is an acronym that

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prove." *Id.* In some proofs, mathematicians prove supporting theorems along the way to proving the main theorem that is the goal of the proof; these supporting theorems are called lemmas. *See* HAMMACK, *supra* note 28, at 88. "A lemma is a theorem whose main purpose is to help prove another theorem." *Id.*

35. *See* HAMMACK, *supra* note 28, at 94 (noting process of generating and writing proof ends by "writ[ing] . . . proof in paragraph form"). "One doesn't normally use a separate line for each sentence in a proof. . . ." *Id.* at 96.

36. Integers are positive and negative whole numbers. *See id.* at 4.

37. This formulation—a statement of the theorem being proven—combines the givens with the thing to be proven. In the standard two-column proof format, the theorem statement could have been rewritten as follows:

Given: x and y are even integers.
Prove: the sum of x and y is even.

38. The proof begins by assuming that the givens are true.

39. This sentence follows from the definition of an even number. If asked to define "even number," most people would likely say that an even number is a number that is divisible by 2. In proofs, mathematicians use a somewhat more formal version of that definition: "An integer n is even if $n = 2a$ for some integer a." *See* HAMMACK, *supra* note 28, at 89. This definition can be useful in proofs because it allows the writer to consider and perform operations on both n and a. Unlike in a two-column proof, where each statement must have a justification, here, some justifications are sufficiently evident—to a knowledgeable mathematical reader—to be omitted. *See id.* at 87. "A proof should be understandable and convincing to anyone who has the requisite background and knowledge. This knowledge includes an understanding of the meanings of the mathematical words, phrases and symbols that occur in the theorem and its proof." *Id.*

40. The proof has established that the sum of x and y is even. It has therefore reached the intended conclusion and is complete.

41. *See infra* notes 47-54 and accompanying text (discussing IRAC and its variations).

stands for Issue, Rule, Application, Conclusion.[42] In the legal writing classroom, IRAC is often taught as an organizational structure for written legal analysis.[43] Similarly, IRAC is also a commonly suggested strategy for organizing law school exam answers.[44] For any given issue, following the acronym in order, students should state what the issue is, state the relevant rules, apply the rules to the facts at hand, and reach a conclusion. That is IRAC in its most basic form.

Figure 3: IRAC

I	The issue is whether Ms. Garcia and Mr. Jordan formed a common-law marriage while in Rhode Island.
R	A common-law marriage is formed when (1) a couple had a serious, mutual present intent to enter a spousal relationship and (2) the actions of the couple led to a belief in the community that they were in a spousal relationship. *Smith v. Smith*, 966 A.2d 109, 114, 117 (R.I. 2009).
A	Here, Ms. Garcia and Mr. Jordan had a mutual present intent to enter a spousal relationship because they promised each other, when they moved in together, that they would be "like husband and wife" from that moment on. Furthermore, their actions led to a belief in the community that they were in a spousal relationship. The seventeen witnesses at trial—family and friends of the couple who knew them well—all testified that they considered Mr. Jordan and Ms. Garcia to be husband and wife.
C	Therefore, Ms. Garcia and Mr. Jordan did form a common-law marriage.

One common variation on the IRAC structure is CRAC (Conclusion, Rule, Application, Conclusion), which replaces the Issue at the beginning of IRAC with a statement of the Conclusion on that issue.[45] This variation is often used for persuasive writing such as appellate briefs.[46] While a statement of the issue is informative, a statement of one's conclusion on that issue is more persuasive.[47] For example, stating that "the issue is whether the parties formed a common-law marriage while in Rhode Island," will let the reader know that the issue is about common-law marriage. More persuasively, however, stating that "the parties

42. *See* CHARLES R. CALLEROS, LEGAL METHOD AND WRITING 71 (5th ed. 2006) (detailing elements of IRAC).

43. *See id.* (demonstrating use of IRAC in law school examination essay).

44. *See, e.g.*, ANN L. IIJIMA, THE LAW STUDENT'S POCKET MENTOR: FROM SURVIVING TO THRIVING 118-20 (2007) (outlining each element of IRAC); ALEX RUSKELL, A WEEKLY GUIDE TO BEING A MODEL LAW STUDENT 86-89 (2015) (explaining elements of IRAC); HELENE S. SHAPO ET AL., WRITING AND ANALYSIS IN THE LAW 453 (W. Acad. 7th ed. 2018) (listing elements of IRAC).

45. *See* CALLEROS, *supra* note 42, at 331 (outlining acronym CRAC in comparison to IRAC).

46. *See id.* (demonstrating use of CRAC in persuasive writing).

47. MICHAEL R. FONTHAM ET AL., PERSUASIVE WRITTEN AND ORAL ADVOCACY IN TRIAL AND APPELLATE COURTS 11 (2d ed. 2007) (highlighting CRAC's superiority in persuasive legal writing).

formed a common-law marriage while in Rhode Island," will focus the reader on the writer's conclusion on the same issue.

Beyond the switch to CRAC for persuasive writing, many IRAC variations attempt to expand the structure in various ways to help the students fully flesh out complex legal arguments. Consider the IRAC example above in Figure 3: The facts to which the rules are applied are very one-sided; the facts are strongly supportive of a common-law marriage and there are no facts that would indicate a common-law marriage had not been established. With other, more equivocal facts, a short IRAC would be insufficient. Consider, for example, if instead of promising to be "husband and wife," the couple had promised to be "together for life." Would that be enough to show the requisite "mutual present intent" to enter a spousal relationship? Maybe, maybe not, but the reader will probably need more information. The writer should look at case law to see whether courts have given more information about what shows "mutual present intent." If two of the witnesses at trial testified that they considered the couple to be dating rather than married, would there still be enough proof of a community belief that they were spouses? Perhaps. The writer should again look at the precedent to see whether previous cases can help answer that question. In order to fully analyze the scenario with more equivocal facts, the writer will need to expand the R and A sections of IRAC.

Over the years, scholars have created new acronyms to help show students how best to use IRAC when the reader will need more detail and explanation. These variations on IRAC include CREAC (Conclusion, Rule, Explanation, Application, Conclusion);[48] CRuPAC (Conclusion, Rule, Proof of Rule, Application, Conclusion);[49] TREAT (Thesis, Rule, Explanation, Application, Thesis);[50] CREXAC (Conclusion, Rule, Explanation, Application, Conclusion);[51] and CRRPAP (Conclusion, Rule, Rule Proof, Application, Prediction).[52] Many of these IRAC variants provide a place for a deeper exploration of the rule at issue. For example, in CREAC, the added E is for "explanation" of the rule.[53] For students new to legal writing, this often centers

48. *See* HEIDI K. BROWN, THE MINDFUL LEGAL WRITER: MASTERING PREDICTIVE WRITING 91 n.1 (2015) (discussing CREAC as alternative to IRAC).

49. *See* RICHARD K. NEUMANN, JR., LEGAL REASONING AND LEGAL WRITING: STRUCTURE, STRATEGY, AND STYLE 94 (6th ed. 2009) (detailing elements of CRuPAC).

50. *See* MICHAEL D. MURRAY & CHRISTY HALLAM DESANCTIS, LEGAL WRITING AND ANALYSIS 139 (2009) (identifying elements of TREAT).

51. *See* MARY BETH BEAZLEY, A PRACTICAL GUIDE TO APPELLATE ADVOCACY 77 (3d ed. 2010) (listing elements of CREXAC).

52. *See* CHRISTINE COUGHLIN ET AL., A LAWYER WRITES: A PRACTICAL GUIDE TO LEGAL ANALYSIS 83 (2d ed. 2013) (defining CRRPAP). Further variations include BaRAC, CRARC, IGPAC, IRAAC(P), IRAAPC, IRAC with EIP, IRAC plus, IRRAC, RAFADC, TREAC, and TRRAC. *See* Tracy Turner, *Finding Consensus in Legal Writing Discourse Regarding Organizational Structure: A Review and Analysis of the Use of IRAC and Its Progenies*, 9 LEGAL COMM. & RHETORIC: JALWD 351, 357-58 (2012) (displaying table of IRAC variant acronyms).

53. *See* BROWN, *supra* note 48, at 91 n.1, 94.

on the legal precedent that will later be used in A. In E, for each precedent case, the writer gives the facts, reasoning, and holding that are relevant to the issue the IRAC addresses.[54] Fleshing out the cases in this way helps the reader see how courts have interpreted and applied the rules in R, and allows the writer to later analogize or distinguish the case at hand and the precedent cases in A.[55] Since the information in E is detailed information about the rule, it could be considered part of R.[56] Many legal writing professors, however, find it useful to include E in the acronym so that students remember to include this information. The information in E is not limited to the facts, reasoning, and holding of precedent. As appropriate, E can be used for other details that help explain the rule. For a statutory rule, for example, E might include legislative history relevant to the issue at hand.

B. *Types of Reasoning*

As noted earlier, lawyers and mathematicians use similar reasoning in their thought processes and writing. In considering problems, both mathematicians and lawyers use deductive reasoning, inductive reasoning, and arguments in the alternative.

1. *Deduction*

At their cores, both mathematical proofs and IRAC use deductive reasoning and syllogisms.[57] The syllogism is a structured form of deductive reasoning—reasoning from the general to the specific—involving three parts: the major premise, which is the general rule; the minor premise, which consists of the specific facts at hand; and the conclusion about the specific facts in light of the general rule.[58] Figure 4, below, shows a well-known syllogism.[59]

Figure 4: Syllogism Example

Step 1.	All humans are mortal.[60]	Major premise
Step 2.	Socrates is human.	Minor premise
Step 3.	Therefore, Socrates is mortal.	Conclusion

54. *See id.* at 112 (addressing use of precedent in explanation section).

55. *See id.* at 146 (discussing ways writer can compare and contrast precedent).

56. The information in E helps establish the "lemmas" of legal writing—the "theorems" that help prove the main "theorem." *See supra* note 34 and accompanying text (defining "lemmas").

57. *See* KRISTEN KONRAD TISCIONE, RHETORIC FOR LEGAL WRITERS: THE THEORY AND PRACTICE OF ANALYSIS AND PERSUASION 88 (2d ed. 2016) (explaining IRAC is "shorthand for deductive reasoning").

58. *See id.* at 87-88 (explaining syllogisms and deductive reasoning).

59. *See id.* at 88.

60. This statement can be reworded to fit the if-then format, also known as a conditional statement: If a thing is human, then that thing is mortal. *See infra* note 172 and accompanying text (explaining conditional statements).

In mathematics, each step of the proof should rely on a syllogism. For example, moving from Steps 3 and 4 to Step 5 in the two-column proof above[61] involved the following syllogism:

Figure 5: Syllogism in Mathematical Proof

The Transitive Property of Equality (for any numbers x, y, and z, if $x = y$ and $y = z$, then $x = z$).	Major premise
The measure of Angle A + the measure of Angle B = 180 degrees and the measure of Angle C + the measure of Angle B = 180 degrees.	Minor premise
Therefore, the measure of Angle A + the measure of Angle B = the measure of Angle C + the measure of Angle B.	Conclusion

Similarly, the organization of IRAC relies on the structure of syllogisms.[62] The RAC of IRAC is a syllogism.[63]

Figure 6: Syllogism in IRAC

A common-law marriage is formed when (1) a couple had a mutual present intent to enter a spousal relationship and (2) the actions of the couple led to a belief in the community that they were in a spousal relationship.[64] *Smith v. Smith*, 966 A.2d 109, 114, 117 (R.I. 2009).	Major premise
Here, Ms. Garcia and Mr. Jordan had a mutual present intent to enter a spousal relationship. Furthermore, their actions led to a belief in the community that they were in a spousal relationship.	Minor premise
Therefore, Ms. Garcia and Mr. Jordan did form a common-law marriage.	Conclusion

61. *See supra* Figure 1.

62. *See* Anita Schnee, *Logical Reasoning "Obviously"*, 3 LEGAL WRITING: J. LEGAL WRITING INST. 105, 106 (1997) (noting "'IRAC' . . . just another way of saying . . . 'deductive syllogistic process'").

63. *See* Nelson P. Miller & Bradley J. Charles, *Meeting the Carnegie Report's Challenge to Make Legal Analysis Explicit—Subsidiary Skills to the IRAC Framework*, 59 J. LEGAL EDUC. 192, 208 (2009) (explaining IRAC "mirrors . . . syllogism"); Turner, *supra* note 52, at 356 (explaining "adaptation of deductive syllogism to legal reasoning"); Penny L. Willrich, *The Path to Resilience: Integrating Critical Thinking Skills into the Family Law Curriculum*, 3 PHX. L. REV. 435, 451 (2010). "[T]he major premise is based upon the applicable rules. The minor premise is found in the facts or evidence." Willrich, *supra*, at 451.

64. This rule can also be reworded as an if-then conditional statement: If a couple had a mutual present intent to enter a spousal relationship and the actions of the couple led to a belief in the community that they were in a spousal relationship, then the couple formed a common-law marriage. *See infra* note 172 and accompanying text (detailing conditional statements).

2. Induction

While deductive reasoning is reasoning from the general to the specific—applying a general rule to specific facts—inductive reasoning is reasoning from the specific to the general—using specific facts to try to find a general rule.[65]

Mathematicians use induction in two different, but related, ways. George Polya, an influential mathematician and mathematics educator who wrote the book *How to Solve It: A New Aspect of Mathematical Method*—which has helped generations of mathematics students understand mathematical reasoning and proof—used different terms for the two types of induction.[66] He distinguished between *induction*, "the process of discovering general laws by the observation and combination of particular instances," and *mathematical induction*, the rigorous method of proof by induction.[67]

In mathematical induction, i.e., a proof by induction, a mathematician uses the fact that a statement is true for one case to show that it must be true for all cases.[68] Proof by induction requires two steps.[69] First, in the "basis step," the writer shows that the statement is true for one specific case, often the number "1."[70] Second, in the "inductive step," the writer first assumes that the statement is true for any positive integer x (the "induction hypothesis"), and then shows, based on that assumption, that the statement must also be true for the integer $x + 1$.[71] If the writer can establish both of those things, then it follows that the statement is true for all positive integers because it is true for "1"; the fact it is true for "1" implies it is true for "2" (based on the inductive step); the fact it is true for "2" implies it is true for "3" (again based on the inductive step); and so on.

A common analogy is to toppling dominoes. In the basis step, the writer shows that the first domino will topple. In the inductive step, the writer shows that toppling one domino will topple the next domino. Putting those two facts together shows that all the dominos will topple. Here is an example of a proof by induction:

65. *See* Schnee, *supra* note 62, at 111 (comparing deductive reasoning and inductive reasoning).

66. GEORGE POLYA, HOW TO SOLVE IT: A NEW ASPECT OF MATHEMATICAL METHOD 114 (Anchor Books 2d ed. 1985).

67. *Id.* (describing types of induction).

68. *See* HAMMACK, *supra* note 28, at 155 (giving example of proof by induction).

69. *See id.* at 156 (outlining proof by induction).

70. *See id.* (detailing basis step of proof by induction).

71. *See id.* (explaining inductive step and induction hypothesis).

Figure 7: Proof by Induction

Theorem: For any positive integer n, the sum of the integers from 1 through n equals $n(n + 1)/2$. In other words, $1 + 2 + 3 + \ldots + n = n(n + 1)/2$.

Proof.

[Basis Step][72] If $n = 1$, then the sum of the integers 1 through n is 1.[73] When $n = 1$, $n(n + 1)/2 = 1(1 + 1)/2 = 1(2)/2 = 2/2 = 1$.[74] Therefore, the statement $1 + 2 + 3 + \ldots + n = n(n + 1)/2$ is true where $n = 1$.[75]

[Inductive Step] Suppose k is a positive integer and that $1 + 2 + 3 + \ldots + k = k(k + 1)/2$.[76] Then this proof must show that $1 + 2 + 3 + \ldots + (k + 1) = (k + 1)((k + 1) + 1)/2 = (k + 1)(k + 2)/2$.

$$1 + 2 + 3 + \ldots + (k + 1) =$$

$$1 + 2 + 3 + \ldots + k + (k + 1) =$$

$$k(k + 1)/2 + (k + 1) =$$ [77]

$$k(k + 1)/2 + 2(k + 1)/2 =$$ [78]

$$(k + 2)(k + 1)/2$$ [79]

Therefore, based on induction, for any positive integer n, the sum of the integers from 1 through n equals $n(n + 1)/2$.

Mathematicians also use inductive reasoning before the stage of setting down a formal proof. While the inductive proof above was proven using mathematical induction, a mathematician could have used inductive reasoning to generate a hypothesis to prove in the first place. The thought process might go like this:

72. For the basis step, the writer needs to show that equation in the theorem ($1 + 2 + 3 + \ldots + n = n(n + 1)/2$) is true when $n = 1$.

73. This takes care of the left side of the equation.

74. And this solves the right side of the equation.

75. This follows from the fact that the previous two steps showed that both sides of the equation equal one when $n = 1$.

76. This is the induction hypothesis. The goal now is to prove that the statement holds true for $(k + 1)$, i.e., $1 + 2 + 3 + \ldots + (k + 1) = (k + 1)((k + 1) + 1)/2$.

77. This step uses the assumption in the induction hypothesis to replace $1 + 2 + 3 + \ldots + k$ with the expression we assumed it was equal to in the induction hypothesis, $k(k + 1)/2$.

78. This step multiplies the expression $(k + 1)$ by $2/2$—otherwise known as 1, meaning that the value does not change—so that the two fractions have a common denominator.

79. This step adds the fractions. Having reduced the expressions, this series of statements has shown that $1 + 2 + 3 + \ldots + (k + 1) = (k + 1)(k + 2)/2$, which was the goal of the inductive step.

Figure 8: Inductive Reasoning
(the thought process of a mathematician)

Is there a general formula for the sum of the first *n* integers? Well, when *n* is 1, the sum of the first *n* integers is just 1. When *n* is 2, the sum of the first 2 integers is *1* + *2* or *3*. Let me look at the first several integers.

n	Sum of the first *n* integers
1	1
2	$1 + 2 = 3$
3	$1 + 2 + 3 = 6$
4	$1 + 2 + 3 + 4 = 10$
5	$1 + 2 + 3 + 4 + 5 = 15$
6	$1 + 2 + 3 + 4 + 5 + 6 = 21$
7	$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$

Well the odd numbers are interesting. The sums are multiples of *n* when *n* is odd, at least for these numbers. Let's look at those multiples.

n	Sum of the first *n* integers	*n* times what?
1	1	1
3	$1 + 2 + 3 = 6$	2
5	$1 + 2 + 3 + 4 + 5 = 15$	3
7	$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$	4

Aha! The multipliers follow a pattern! In fact, the even numbers fit the pattern, too, it's just that the multipliers are not whole numbers.

n	Sum of the first *n* intergers	*n* times what?
1	1	1
2	$1 + 2 = 3$	1.5
3	$1 + 2 + 3 = 6$	2
4	$1 + 2 + 3 + 4 = 10$	2.5
5	$1 + 2 + 3 + 4 + 5 = 15$	3
6	$1 + 2 + 3 + 4 + 5 + 6 = 21$	3.5
7	$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$	4

So, then, can this be translated into a general formula? It appears, for each *n*, the multiplier is half of the next *n* on the list. So maybe it is true for any integer *n* that the sum of the integers from 1 through *n* equals $n(n + 1)/2$.

The inductive thought process yields a possible formula for the sum of the first *n* integers, but it does not prove that the formula always holds true. It is possible that the pattern will break down when *n* equals 8, or 1,008, or some other number. As shown in the proof in Figure 7, however, mathematical induction can be used to prove that the formula is correct and applies not just to the first seven integers, but to all positive integers.

Although lawyers do not create domino-toppling proofs by induction, they do use inductive reasoning much like the mathematical thought process above. Lawyers and mathematicians alike can use induction—as distinct from mathematical induction—to try to identify general rules based on specific factual scenarios. While mathematicians use inductive reasoning based on specific numbers, lawyers use inductive reasoning on the facts of specific cases.

For lawyers, inductive reasoning works hand-in-hand with the common law and the use of precedent. Lawyers consider the facts of earlier cases to try to find a pattern that might indicate a general principle of law that could be relevant to the case at hand.[80] For example, a lawyer analyzing a common-law marriage issue[81] might come upon cases with the following facts relating to the community belief:

> Case One: Five witnesses testified, including two family members of the putative spouses. All five stated that they thought the couple was married even though the couple used different last names. The court held that there was a community belief that the couple was married.
>
> Case Two: Three witnesses testified. One of the witnesses, the putative husband's brother, stated that his family—including the putative husband—treated the woman as a girlfriend rather than a spouse. The brother also testified that his great-aunt, the family's unofficial genealogist, did not include the woman in the family tree she had created six months before. The other two witnesses, both friends of the couple, stated that they believed the couple to be married. The court held that there was no community belief that the couple was married.
>
> Case Three: Ten witnesses testified. All ten stated that they believed the couple to be married and knew the woman by the man's last name. The court held that there was a community belief that the couple was married.
>
> Case Four: Four witnesses testified. The first witness, the couple's upstairs neighbor and landlord, stated that she thought the couple was married. The other three witnesses, friends of the couple, testified that they thought of the couple as in a long-term relationship but not a marriage. The court held that there was no community belief that the couple was married.

A lawyer, reading these four cases, could use inductive reasoning to put together the facts of the cases and write the following in a memo or brief:

> Where the testimony and evidence is consistent that community members thought a couple to be married, courts have held that there is a belief in the community that a couple is married. In contrast, when witnesses disagree as to

80. *See* TISCIONE, *supra* note 57, at 80.

81. *Cf. supra* Figures 3, 6 (providing IRAC and syllogism in IRAC regarding common-law marriage issue).

whether a couple is married, courts have held that there is no belief in the community that the couple is married.

The lawyer could then compare the facts of these four cases to the facts of Ms. Garcia and Mr. Jordan's case to analyze how the case should be resolved. If, in the Garcia/Jordan case, all seventeen witnesses at trial testified that they considered Ms. Garcia and Mr. Jordan to be husband and wife, the lawyer could conclude that a court would likely hold that there is a community belief in their spousal relationship.

3. Arguments in the Alternative

One variation on the standard mathematical proof is a proof that uses "cases." In some situations, it is not possible to write a proof that establishes the truth of a statement in all conditions. Instead, the writer of the proof must consider two (or more) cases and prove the statement for each of the cases.[82] For example, the cases might be even integers and odd integers.[83] If the writer can show that a certain statement is true for all even integers and also true for all odd integers, then the writer has proven the statement to be true for all integers. In developing a proof using cases, the writer must be careful in selecting the cases to cover all the possibilities. Since all integers must be either odd or even, separately proving the "odd integers" case and the "even integers" case will validly prove that the statement is true for all integers. If, on the other hand, the writer wanted to prove that a statement was true for all real numbers, proving that it was true for all odd and even integers would not be sufficient because the set of real numbers contains non-integer numbers such as *pi*, ¾, and -19.7.

Legal writers use similar reasoning to argue based on multiple "cases," though the terminology is different. Lawyers argue different "cases" when they make arguments in the alternative. Consider the following excerpt from a table of contents of a federal appellate brief:

ARGUMENT

. . . .

C. Rodriguez Has Not Met His Burden of Establishing the Existence of a Plea Agreement

D. In the Alternative, Even If the Putative Plea Agreement is Assumed to Exist, *Arguendo*, Rodriguez Has Not Met His Burden of Demonstrating that the Government Breached That Agreement In Any Way

. . . .

82. *See* HAMMACK, *supra* note 28, at 98. "In proving a statement is true, we sometimes have to examine multiple cases before showing the statement is true in all possible scenarios." *Id.*

83. *See id.*

G. In the Alternative, Even If the Putative Plea Agreement is Assumed to Exist, *Arguendo*, Rodriguez Has Not Satisfied the Fourth Prong of Plain Error Review[84]

Essentially, the argument in this brief considers two "cases": there is no plea agreement, and there is a plea agreement.[85] In section C, the writer argues that the court should affirm the lower court because there was no plea agreement (the first "case").[86] In sections D through G, the writer argues that the court should affirm the lower court even if there was a plea agreement (the second "case").[87]

4. Reductio Ad Absurdum

Another common tool in the mathematician's toolkit is the proof by contradiction. This type of proof begins with an assumption that the conjecture being proven is false.[88] Based on this assumption, the writer of the proof moves from step to step using theorems, axioms, and deductive reasoning, trying to reach a contradiction, an absurd result.[89] If the steps of the proof do lead to a contradiction, this means that the original assumption—that the conjecture was false—must itself be false, and therefore the conjecture is true.[90]

Figure 9: Proof by Contradiction

Theorem: For any integer n, if n^2 is odd then n is also odd.

Proof. Suppose, for the sake of contradiction, that n^2 is odd and n is even. If n is even, then there exists an integer k such that $n = 2k$.[91] Then:

$$n^2 =$$
$$n * n =$$
$$2k * 2k =$$
$$4(k * k) =$$
$$2(2k * k) =$$

$(2k * k)$ is an integer because it is the product of integers. Therefore, based on the definition of even numbers, n^2 is even since $n * n = 2(2k * k)$. This is a contradiction since we began by supposing that n^2 is odd. Therefore, the supposition is false and the theorem must be true.

84. Brief for Plaintiff-Appellee at iii, United States v. Rodriguez-Diaz, 694 F. App'x 263 (5th Cir. 2017) (No. 16-41300), 2017 WL 1047797, at *iii.

85. *See id.* at 19-20, 23 (outlining main arguments).

86. *See id.* at 19-20 (arguing defendant failed to meet his burden of establishing existence of plea agreement).

87. *See id.* at 20-28 (making alternative arguments).

88. *See* HAMMACK, *supra* note 28, at 111 (describing method of proof by contradiction).

89. *See id.* (showing example of proof by contradiction).

90. *See id.* (describing conclusions that follow from a contradiction).

91. *See supra* note 38 and accompanying text.

In the law, writers also sometimes reason by assuming the opposite of what they are trying to show. For example, in *Corley v. United States*,[92] the Supreme Court rejected the government's argument about a statutory section by, among other things, using *reductio ad absurdum*.[93] The Court noted, if the statute was read literally, as urged by the Government, absurd results would follow.

> Thus would many a Rule of Evidence be overridden in case after case: a defendant's self-incriminating statement to his lawyer would be admissible despite his insistence on attorney-client privilege; a fourth-hand hearsay statement the defendant allegedly made would come in; and a defendant's confession to an entirely unrelated crime committed years earlier would be admissible without more. These are some of the absurdities of literalism that show that Congress could not have been writing in a literalistic frame of mind.[94]

C. *Purposes of the Analysis*

Lawyers and mathematicians use written analysis for similar reasons. Both legal analysis and mathematical analysis help with the writer's thought process, help convince the reader that the writer is correct, help expand the body of knowledge in the field, and help educate the next generation.

1. *Thinking*

The process of writing a proof can help the mathematician think through the problem: "Every mathematician knows that when he/she writes out a proof, new insights, ideas, and questions emerge."[95] At the beginning of the process, the writer has a conjecture as to what the answer should be. Only by working through a proof and finding support for each step of the analysis can the mathematician actually prove that result. This process of finding a method of proving something is not necessarily linear or easy.[96] One path that seems fruitful may ultimately turn out to be a dead end. The person working on the

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92. 556 U.S. 303 (2009).

93. *See id.* at 317. The issue in *Corley* was whether petitioner's written confession to armed bank robbery and conspiracy to commit armed bank robbery was admissible under 18 U.S.C. § 3501, despite the fact he was held for an unnecessary and unreasonable period of time before being taken in front of a magistrate judge. *See id.* at 312-14; *see also* 18 U.S.C. § 3501(a)-(c) (2018) (mandating confessions admissible when voluntary, and not inadmissible solely because of delay).

94. *Corley*, 556 U.S. at 317 (declining to read statute literally).

95. *See* Joseph Auslander, *On the Roles of Proof in Mathematics*, *in* PROOF AND OTHER DILEMMAS: MATHEMATICS AND PHILOSOPHY 61, 66 (Roger A. Simons & Bonnie Gold eds., 2011); *see also* Claudio Bernardi, *What Mathematical Logic Says About the Foundation of Mathematics*, *in* FROM A HEURISTIC POINT OF VIEW: ESSAYS IN HONOUR OF CARLO CELLUCCI 41, 44 n.2 (Cesare Cozzo & Emiliano Ippoliti eds., 2014) ("very often . . . proof[s] allow[] for . . . deeper understanding of . . . subject[s]").

96. *See* Bernardi, *supra* note 95, at 44 (noting difficulties encountered by mathematicians). "[I]n mathematical experience, when checking a method, testing a tool, or hoping that an application will follow, there are very often trials and failures." *Id.*

proof then has to back up and try a different tactic, continuing to reason through the problem, but now in a different way. While trying to fill the gap in his proof of Fermat's Last Theorem, Wiles collaborated with his former student, Richard Taylor.[97] Wiles became convinced that the method they were trying to use would not work, but Taylor was not yet certain.[98] As Wiles thought about why the method was not working, he suddenly realized that "what was making it not work was exactly what would make a method [he had] tried three years before work."[99] By continuing to try to work through the problem, Wiles was able to gain insight into the solution in a way he could never have predicted at the outset.

For lawyers, too, writing is thinking.[100] The process of writing out a legal argument can help the student—or the practitioner or adjudicator—think through the legal problem, grapple with any weaknesses, and arrive at a solution. While a legal writer may have an intuitive sense of the "right" answer, the process of articulating the argument thoroughly and carefully can help the writer either confirm that answer or determine that the result should be different. When the writer is an advocate who must argue for a particular result, the writing process can help identify weak points and claims that will not work so that the advocate can rebut counterarguments, shore up stronger arguments, and, if necessary, concede claims and arguments that are not supportable.

2. Convincing

Another purpose of a mathematical proof is to convince the reader that the analysis is correct.[101] If the writer has shown the reader how known rules apply to the givens, using sound reasoning at each step of the proof, then the reader should agree with the writer that the theorem has been proven.[102]

When Wiles completed his proof of Fermat's Last Theorem, he had to use the proof to convince his readers that each step of the proof was correct and that, therefore, the theorem was true.[103] During the peer review process, the referees

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97. *See* Kolata, *supra* note 14 (describing Wiles's work with Taylor).

98. *See id.*

99. *Id.*

100. *See, e.g.*, George D. Gopen, *CCISSR: The Perfect Way to Teach Legal Writing*, 13 LEGAL WRITING: J. LEGAL WRITING INST. 315, 322 (2007) ("Writing is thinking; thinking is writing."); Roger J. Traynor, *Some Open Questions on the Work of State Appellate Courts*, 24 U. CHI. L. REV. 211, 218 (1957) ("I have not found a better test for the solution of a case than its articulation in writing, which is thinking at its hardest."); Joseph Kimble, *On Legal-Writing Programs*, PERSPS.: TEACHING LEGAL RES. & WRITING , Winter/Spring, 1994, at 44 WL 2 No. 2/3 PERSPEC 43 ("[W]riting is thinking. Thinking on paper. Thinking made visible.").

101. *See* HAMMACK, *supra* note 28, at 89; *see also* CLAUDI ALSINA & ROGER B. NELSEN, CHARMING PROOFS: A JOURNEY INTO ELEGANT MATHEMATICS, at xix (2010). "A proof of a theorem should be absolutely convincing." *See* HAMMACK, *supra* note 28, at 89. "[A] proof is an argument to convince the reader that a mathematical statement must be true." ALSINA & NELSON, *supra*, at xix.

102. *See* REUBEN HERSH, WHAT IS MATHEMATICS, REALLY? 63 (1997). "Practical mathematical proof is what we do to make each other believe our theorems. It's argument that convinces the qualified, skeptical expert." *Id.*

103. *See* SINGH, *supra* note 2, at 33 (reiterating need for rigorous independent review of Wiles's proof).

periodically asked Wiles for further explanation when they came across portions of the proof that they did not understand or that they felt were incomplete.[104] Each time, after further discussion with Wiles, the referees were able to get enough clarification to move on with their review of the proof.[105] Until, one day, Wiles's explanation of the reasons behind a certain assertion was not convincing enough.[106] Although both Wiles and the referee believed the assertion to be true, Wiles realized his proof was incomplete and he went back to the drawing board to bridge the gap, to make sure the entire proof was convincing.[107]

In legal writing, too, the writer is trying to convince the reader that the analysis is correct. Most introductory legal writing texts and courses contain a basic divide between predictive and persuasive legal writing. In predictive writing—exemplified by the office memorandum—the writer attempts to analyze the law objectively and predict the most likely outcome based on the facts of the case and the existing law.[108] In persuasive writing—exemplified by the trial or appellate brief—the writer attempts to advocate for the outcome that will lead to the best result for the client.[109] Considering "persuasion" more broadly, however, both types of legal writing are persuasive. Even with the objective memo, the writer wants to convince the reader that the analysis is correct.[110] If the writer has presented a complete picture of the facts and the law, and shown the reader how the law applies to the facts using sound reasoning, the reader should agree with the writer's conclusions. A judicial opinion, too, is a persuasive document in this way. In writing an opinion, a judge seeks to justify the decision using convincing legal reasoning.

3. *Expanding Knowledge*

Mathematicians also use proofs to expand the realm of mathematical knowledge by establishing the truth of previously unproven mathematical assertions. Though a mathematician may feel a hypothesis is likely true, until there is a valid proof of that hypothesis, the mathematician cannot rely on the hypothesis. Once the mathematician understands a valid proof of the hypothesis, however, the mathematician can then believe in and rely on the result. Moreover, once a proof has been published in a peer-reviewed journal, then "the result is presumed correct, unless there is a compelling reason to believe otherwise."[111] The new theorem is incorporated into the universe of mathematical knowledge

104. *See id.* at 256-57 (recounting email exchanges between referees and Wiles).

105. *See id.* at 256.

106. *See id.* at 259 (describing referee's persistence in asking Wiles for further explanation).

107. *See* SINGH, *supra* note 2, at 257-58 (relating discovery of error).

108. *See, e.g.*, SHAPO ET AL., *supra* note 44, at 149 (claiming legal memorandum "one of . . . most common forms of legal writing").

109. *See id.* at 319.

110. *See id.* at 149 (explaining purpose of objective memorandum).

111. *See* Auslander, *supra* note 95, at 65.

that can be used and built upon by mathematicians the world over, even by mathematicians who have not personally read and understood the proof.[112]

The proof of Fermat's Last Theorem was complex—so complex that it took months to validate and only a handful of mathematicians had the right combination of specialized knowledge to understand the proof. Nonetheless, once the proof was validated and published, mathematicians accepted the result. While the proof is complex, the theorem itself is simple: For any positive integer n greater than 2, there are no whole numbers a, b, and c that make the equation true $a^n + b^n = c^n$. Even someone who is not able to understand the proof of Fermat's Last Theorem can understand the theorem itself and can therefore use the theorem in other proofs.

Publication by peer review is, of course, not a fool-proof method of determining the validity of a proof. Incorrect proofs of valid results—and incorrect proofs of incorrect results—have been published and relied on.[113] Furthermore, even when the proof and its result are correct, "applying a result mechanically, without an understanding of the proof, can lead to errors."[114] In fact, the problem with Wiles's proof of Fermat's Last Theorem—the gap it took him a year to fill—occurred in a portion of the proof where he relied on a colleague's method that he "didn't feel completely comfortable with."[115]

The analogy here in the field of law is not to memos and briefs but rather to judicial opinions. Just as publication of a proof allows mathematicians to make use of the result, once a court has ruled on a case and issued an opinion, the holding is incorporated into the universe of legal knowledge and lawyers can rely on the opinion to make new arguments. As with a peer-reviewed and published proof, precautions apply. Courts can—and do—make mistakes, so the law as stated in a court's opinion is not necessarily correct. Furthermore, as time passes, a "correct" result may be overturned by a later court. Lawyers must also be careful to understand a court's reasoning before attempting to use language from an opinion. Just as "applying a [mathematical] result mechanically, without an understanding of the proof, can lead to errors," so too can applying a court's holding mechanically lead to mistakes by a lawyer.[116]

4. Teaching

In mathematics courses, students engage in proof as part of the learning process. Mathematics students, even at the college level, are not expected to

112. *See id.* (explaining certification of proofs "allows [mathematicians] to use it in further research").

113. *See id.*; Reuben Hersh, *To Establish New Mathematics, We Use Mental Models and Build on Established Mathematics*, *in* FROM A HEURISTIC POINT OF VIEW: ESSAYS IN HONOUR OF CARLO CELLUCCI, *supra* note 95, at 127, 137 (recounting examples of mistakes needing correction).

114. Auslander, *supra* note 95, at 65. "[A] mathematician is not absolved from understanding the proof, even when the result in question has been accepted by the mathematical community." *Id.*

115. *See* Kolata, *supra* note 14 (recounting struggle to solve Fermat's Last Theorem).

116. *See* Auslander, *supra* note 95, at 65 (referring to proofs in mathematics).

establish new theorems never before proven. Instead, they prove theorems that have been understood by mathematicians for hundreds or even thousands of years. The purpose of this is not for the end result of the proof, but for the pedagogical value of the process. The reasoning process—moving step by step through the syllogisms to reach a solution to the problem at hand—is critical to engaging in upper-level mathematics.[117] "Theorems and their proofs lie at the heart of mathematics."[118] Thus, when students learn mathematical proof, they are improving their understanding of mathematics and learning to think like a mathematician.[119]

Similarly, it is a cliché that law school teaches students to "think like a lawyer."[120] IRAC is a large part of that. Identifying legal issues, knowing and understanding legal rules, and being able to apply those rules to facts are all important pieces of "thinking like a lawyer."[121]

III. BUT DOESN'T LAW HAVE LESS CERTAINTY THAN MATHEMATICS?

"[T]he moment you leave the path of merely logical deduction you lose the illusion of certainty which makes legal reasoning seem like mathematics. But the certainty is only an illusion, nevertheless."[122]

"Absolute certainty is what many yearn for in childhood, but learn to live without in adult life, including in mathematics."[123]

Comparisons between legal reasoning and mathematical reasoning are not new, nor are criticisms of such comparisons.[124] Langdell famously wanted to treat law as a science and developed his casebook method at Harvard around that

117. *See* John H. Conway, *Foreword* to HOW TO SOLVE IT: A NEW ASPECT OF MATHEMATICAL METHOD, *supra* note 66, at xx. "Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean [to be] doing mathematics? In the first place, it means to be able to solve mathematical problems." *Id.* (quoting George Polya).

118. *See* ALSINA & NELSON, *supra* note 101, at ix.

119. *See* WILLIAM BYERS, DEEP THINKING: WHAT MATHEMATICS CAN TEACH US ABOUT THE MIND 106 (2015) ("The formal theorems and proofs deepen our understanding of the concepts involved. Conversely a deeper conceptual understanding leads to new and better theorems.").

120. *See* TISCIONE, *supra* note 57, at 1 (noting common analytical thinking development among law students).

121. *See id.*

122. Oliver Wendell Holmes, Jr., *Privilege, Malice, and Intent*, 8 HARV. L. REV. 1, 7 (1894).

123. RUEBEN HERSH, EXPERIENCING MATHEMATICS: WHAT DO WE DO, WHEN WE DO MATHEMATICS? 81 (2014).

124. *See, e.g.*, Morris R. Cohen, *The Place of Logic in the Law*, 29 HARV. L. REV. 622, 624 (1916).

> We try to reduce the law to the smallest number of general principles from which all possible cases can be reached, just as we try to reduce our knowledge of nature to a deductive mathematical system. . . .
>
> The law, of course, never succeeds in becoming a completely deductive system. It does not even succeed in becoming completely consistent.

Id.

idea.[125] Holmes agreed that law was a science,[126] but maintained that law and mathematics cannot be equated.[127] When the attempts failed, the resulting insights into the nature of mathematics were truly groundbreaking. The first insight came in the area of geometry. For more than a thousand years, following in Euclid's footsteps, mathematicians studied geometry based on five postulates:

1. For any two points, a straight line segment can be drawn to join those two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. For any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. If two straight lines intersect a third straight line in such a way that the sum of the inner angles on one side of the intersection line is less than two right angles, then the two lines must intersect each other on that side if extended indefinitely.[128]

In his *Elements*, Euclid built on these five postulates to prove theorems and explain geometry.[129] The first four statements seemed self-evident and therefore were properly considered postulates because they were true but could not be proven.[130] The fifth postulate, however, caused some trouble because of a sense that it was not as self-evident as the other postulates and therefore should be proven rather than merely assumed to be true. Euclid was "reluctant" to introduce it, and mathematicians who followed Euclid attempted to prove the fifth postulate, all without success.[131]

The insight, when it eventually came many centuries after Euclid, was that the fifth postulate is not necessarily true. One method mathematicians tried to use to prove the fifth postulate was proof by contradiction.[132] Though a fruitless effort, one of those mathematicians who attempted it, instead of giving up

125. *See, e.g.*, Gerald P. Lopez, *Transform—Don't Just Tinker with—Legal Education*, 23 CLINICAL L. REV. 471, 517 (2017) (describing Langell's teaching methods).

126. Oliver Wendell Holmes, *Law in Science and Science in Law*, 12 HARV. L. REV. 443, 452 (1899). "The true science of the law does not consist mainly in a theological working out of dogma or a logical development as in mathematics . . . ; an even more important part consists in the establishment of its postulates from within upon accurately measured social desires instead of tradition." *Id.*

127. *See* Oliver Wendell Holmes, *The Path of the Law*, 10 HARV. L. REV. 457, 465 (1897). "The danger of which I speak is . . . the notion that a given system, ours, for instance, can be worked out like mathematics from some general axioms of conduct." *Id.*

128. *See* COXETER, *supra* note 21, at 1 (listing Euclid's five postulates).

129. *See* Gray, *supra* note 21, at 83-84 (describing organization of Euclid's *Elements*).

130. *See* COXETER, *supra* note 21, at 2 (contrasting first four postulates with fifth because "not self-evident like . . . others").

131. *See id.* (explaining distinction between postulate five and other four postulates).

132. *See* MEYER, *supra* note 31, at 106-07.

because he failed to prove the fifth postulate, kept delving deeper into the insights he gained when he assumed that the fifth postulate was false.[133] In doing so, he—along with a few other mathematicians who had similar insights in the same time period—discovered hyperbolic geometry, which is now considered one of two main types of non-Euclidean geometry.[134] While the fifth postulate is true of lines and points on a flat plane, it is not true in hyperbolic geometry, which is the geometry of a saddle-shaped surface.

For thousands of years, mathematicians assumed that Euclid's postulates were true. But with the discovery of non-Euclidean geometry, it became clear that they may or may not be true depending on the system in which one was operating. Furthermore, different systems can be built upon different axioms. Mathematicians developed hyperbolic geometry based on a different set of axioms, only some of which are the same as Euclid's postulates.[135] Elliptic geometry, the other main type of non-Euclidean geometry, is built on yet another set of axioms.[136]

In the early 1900s, mathematician David Hilbert was working on an axiomatization of mathematics.[137] That is, he wanted to develop formal systems of mathematics starting from a small number of basic axioms, much as geometry is built upon a foundation of axioms.[138] Importantly, Hilbert, who is often described as a formalist, wanted to show that each formal system was complete and consistent.[139] A system is complete if any statement within that system can be proven to be either true or false.[140] A system is consistent if, using the axioms of that system, it is not possible to prove a statement both true and false.[141] While Hilbert was attempting to develop these formal systems, another mathematician announced a proof that suggested Hilbert would never succeed.

Kurt Gödel, a young mathematician who had only recently finished his dissertation, wanted to support Hilbert's goals by proving the consistency of a subset of mathematics known as analysis.[142] Instead, Gödel's reasoning led to

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133. *See id.* at 107; *see also* COXETER, *supra* note 21, at 5.

134. *See* COXETER, *supra* note 21, at 4-5 (describing development of non-Euclidean hyperbolic geometry through attempts to prove parallel postulate). Coxeter noted "non-Euclidean geometry" typically refers to hyperbolic and elliptic geometry. *Id.*

135. *See* MEYER, *supra* note 31, at 102 (defining hyperbolic geometry).

136. *See id.* at 95 (defining elliptic geometry).

137. *See* Donald Gillies, *Serendipity and Mathematical Logic*, *in* FROM A HEURISTIC POINT OF VIEW: ESSAYS IN HONOUR OF CARLO CELLUCCI, *supra* note 95, at 23, 25. In Hilbert's version of formalism, "mathematics consisted of a collection of formal systems in each of which the theorems were deduced from the axioms using mathematical logic." *Id.*

138. *See id.*

139. *See id.*; *see also* Stephen C. Kleene, *Introductory Note to 1930b, 1931 and 1932b*, *in* 1 KURT GÖDEL: COLLECTED WORKS 126, 127 (Solomon Feferman et al. eds., 1986) (discussing Hilbert's proposals).

140. *See* RON AHARONI, MATHEMATICS, POETRY AND BEAUTY 135 (2015) (describing Hilbert's goal of proving completeness for system of axioms); Kleene, *supra* note 139, at 127.

141. *See* AHARONI, *supra* note 140, at 135 (describing Hilbert's goal of proving consistency for system of axioms); *see also* Kleene, *supra* note 139, at 127 (explaining Hilbert's consistency of systems requirement).

142. *See* Kleene, *supra* note 139, at 127.

his Incompleteness Theorem, which called into question the attempts at mathematical formalism.[143] Gödel was inspired in part by the ancient Liar's Paradox,[144] which is the paradox that results from trying to decide whether the following sentence is true or false: This sentence is false. If the statement "this sentence is false" is true, then by its own terms, it is false. And if the statement "this sentence is false" is false, then, because it says it is false, it must be true. Either route leads to a contradiction and it is not possible to call the sentence either true or false.

Gödel's proof extended this idea to the concept of provability.[145] Gödel proved that if a formal system of mathematics is consistent, then there is a statement within that system that is not provable, meaning that there is no proof that the statement is true and also no proof that the statement is false.[146] "Gödel's incompleteness theorem implies that there can be no formal system that is consistent, yet powerful enough to serve as a basis for all of the mathematics that we do."[147]

These insights do not change the fact that mathematical reasoning and mathematical proof are useful. Mathematicians continue to theorize about mathematics and prove theorems, though they now do so with an awareness that the axioms on which the theorems are built are merely assumptions—a working model for one view of a mathematical universe—and that the model itself cannot be complete.[148] Similarly, in the law, good, solid reasoning is useful even though lawyers should be aware that the system of laws is never complete.

In attempting to describe commonalities among legal realist thinkers of his day, Llewellyn identified several points, including the following: a "conception of law in flux"; a "conception of society in flux . . . so that the probability is always given that any portion of law needs reexamination to determine how far it fits the society it purports to serve"; a "[d]istrust of traditional legal rules and concepts insofar as they purport to *describe* what either courts or people are actually doing"; and a "belief in the worthwhileness of grouping cases and legal

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143. *See id.* at 127-28.

144. *See* Kurt Gödel, *On Formally Undecidable Propositions of Principia Mathematica and Related Systems I*, *in* 1 KURT GÖDEL: COLLECTED WORKS, *supra* note 139, 145, 149-51.

145. *See id.* (laying out "a proposition that says about itself that it is not provable").

146. *See* Bernardi, *supra* note 95, at 26 (summarizing the result of Gödel's First Incompleteness Theorem).

147. William P. Thurston, *On Proof and Progress in Mathematics*, 30 BULL. AM. MATHEMATICAL SOC'Y. 161, 170 (1994).

148. *See* RICHARD L. EPSTEIN, REASONING IN SCIENCE AND MATHEMATICS: ESSAYS ON LOGIC AS THE ART OF REASONING WELL 77 (2012) (highlighting assumptions underlying mathematical proofs). Some philosophers of mathematics even argue that proofs built upon axioms do not truly prove anything since the axioms themselves "are not established, they are simply postulated." Hersh, *supra* note 113, at 134. "Perhaps in the nineteenth century, logic was regarded as a way to guarantee the certainty of mathematics. But nowadays we do not expect that much: [I]t seems naïve, and perhaps even futile, to hope for a definitive, proven certainty of mathematics." Bernardi, *supra* note 95, at 42.

situations into narrower categories than has been the practice in the past."[149] All of these points reveal similarities between law and mathematics rather than differences.

Beginning with the last point, "grouping cases and legal situations into narrower categories," i.e., creating narrower rules, does not change the reasoning process necessary to apply rules—whether broad or narrow—to the facts at hand. Indeed, Llewellyn thought logic had a place in legal reasoning, though he believed that other aspects were important as well.[150]

Llewellyn also noted that legal realists conceive of both law and the society to which law should respond as "in flux." [151] Similarly, mathematicians now acknowledge that the field of mathematics changes and that it is a function of its time and place.[152] As philosopher of mathematics Reuben Hersh has observed, "[t]he body of established mathematics is not a fixed or static set of statements. . . . Established mathematics is established on the basis of history, social practice, and internal coherence. . . . What has been published remains subject to criticism or correction."[153]

The legal realists' "[d]istrust of traditional legal rules and concepts insofar as they purport to describe what either courts or people are actually doing,"[154] also has echoes in mathematics. "A mathematical theorem does not show that a claim is true. It shows that the claim follows from the assumptions of the theory."[155]

The Greek word for "axiom" originated not in mathematics, but in rhetoric.[156] Originally, it meant a statement that was assumed to be true for purposes of the

149. *See* Karl N. Llewellyn, *Some Realism about Realism—Responding to Dean Pound*, 44 HARV. L. REV. 1222, 1235-37 (1931) (describing "common points of departure" of those in legal realism movement).

150. *See* Karl N. Llewellyn, *A Realistic Jurisprudence—The Next Step*, 30 COLUM. L. REV. 431, 447 n.12 (1930).

> A careful study of the formal logic of judicial opinions would be a useful study. But I would urge that even its usefulness would be hugely increased by an equally careful study of the instrumentalism, the pragmatic and socio-psychological decision elements in the same cases. And that an equally geometric increase in illumination would follow a further careful study of the effects on the society concerned of the same cases.

Id.

151. *See* Llewellyn, *supra* note 149, at 1236.

152. *See* Michael D. Resnick, *Proof as a Source of Truth*, *in* PROOF AND KNOWLEDGE IN MATHEMATICS 1, 7 (Michael Detlefsen ed., 1992) (considering "proofs . . . social and cultural objects"). Proofs "evolve in a particular social and cultural context, and they have intended audiences." *Id.*

153. *See* Hersh, *supra* note 113, at 132. "A warrant is a justification to act on an assertion, a justification based on lived experience." *Id.* at 137.

154. *See* Llewellyn, *supra* note 149, at 1237.

155. *See* EPSTEIN, *supra* note 148, at 77; *see also* Thurston, *supra* note 147, at 170 ("[m]ost mathematicians adhere to foundational principles . . . known to be polite fictions.").

156. *See* Hans Niels Jahnke, *The Conjoint Origin of Proof and Theoretical Physics*, *in* THE BEST WRITING ON MATHEMATICS 2011, at 236, 237-38 (Mircea Pitici ed., 2012) (detailing linguistic history of mathematical terms).

rhetorical argument.[157] The absolute truth of the axiom was not necessary, just that those arguing agreed to take the statement as true.[158] Around the same time the term was imported into mathematics, it came to have its current meaning: a statement that mathematicians believe to be true but that cannot be proved.[159] With the insights of non-Euclidean geometry and Gödel's Incompleteness Theorem, mathematicians' understanding of the word "axiom" is closer to its original meaning: something that we take as true for the sake of argument.[160] In the legal field, lawyers treat rules in much the same way. Lawyers take the laws and rules to be "true" and form their arguments based on those rules.

IV. Can Anything Account for the Similarities?

The work and theories of cognitive psychologists and other cognitive scientists may help explain the similarities between mathematical analysis and legal analysis. Many of the shared features help improve the "fluency" of the writing.

With both mathematical proofs and legal writing, communication is an important function of the document. For the document to serve its purpose, the reader of the document must understand it and believe it. The "fluency" of the document is key. Fluency is "a subjective experience of ease or difficulty associated with a mental process. In other words, fluency isn't the process itself but, rather, information about how efficient or easy that process feels."[161] Fluent documents, to the reader, seem to be easier to process than disfluent documents. Importantly, for mathematical analysis and for legal analysis, when a document is more fluent, the reader is more likely to perceive the document as true and to have higher confidence in the document.[162] Since legal writers and

157. *See id.* at 238 (explaining origin of "axiom" terminology).

158. *See id.* at 238-39.

159. *See id.* (describing shift to current meaning of axiom).

160. *See* Jahnke, *supra* note 156, at 247-48.

> [U]ntil the end of the nineteenth century, mathematicians were convinced that mathematics rest on intuitively secure intrinsic hypotheses which determined the inner identity of mathematics. . . . Then, non-Euclidian geometries were discovered. The subsequent discussions about the foundations of mathematics at the beginning of the twentieth century resulted in the decisive insight that pure mathematics cannot exist without hypotheses (axioms) which can only be justified extrinsically.

Id.

161. *See* Daniel M. Oppenheimer, *The Secret Life of Fluency*, 12 TRENDS COGNITIVE SCI. 237, 238 (2008) (explaining role of fluency in judgments and cognition).

162. *See* Daniel M. Oppenheimer, *Consequences of Erudite Vernacular Utilized Irrespective of Necessity: Problems with Using Long Words Needlessly*, 20 APPLIED COGNITIVE PSYCHOL. 139, 140 (2006) (explaining processing "[f]luency leads to higher [j]udgements of truth, confidence, . . . and even liking"); *see also* Julie A. Baker, *And the Winner Is: How Principles of Cognitive Science Resolve the Plain Language Debate*, 80 UMKC L. REV. 287, 288 (2011). Research shows that "the more 'fluent' a piece of written information is, the better a reader will understand it, and the better he or she will like, trust and believe it." Baker, *supra*, at 288.

mathematicians alike want their readers to believe and have confidence in their analyses, more fluent documents should be more successful documents.

Familiarity breeds fluency, so reading the same phrase repeatedly increases fluency, as does seeing something presented in an expected way.[163] Therefore, even if readers respond to IRAC or to the organizational structure of mathematical proof in part because that's the way it has always been done, the familiarity of the organizational scheme is nonetheless helpful to the reader. Furthermore, as Professor Lucille A. Jewell has cogently argued, the syllogism has endured in part because it promotes fluency.[164] The fluency of a syllogism relates to familiarity.[165] The repetition of terms from the major and minor premise to the conclusion increases the familiarity of those terms.[166]

The structure of sentences and paragraphs can also increase fluency. Linguists have observed that sentences are often composed of "given" information (the information the reader already knows), and "new" information (the information the writer is introducing for the first time in that sentence).[167] Furthermore, in English, sentences often have the given information in the first part of the sentence and the new information in the second part of the sentence.[168] Readers find passages more comprehensible when sentences are structured this way.[169] In a paragraph, the sentences can be structured so that each sentence follows from the previous sentence such that the new information in one sentence becomes the given information in the next sentence.[170]

Sentence 1: [given information A] [new information B]

Sentence 2: [given information B] [new information C]

Sentence 3: [given information C] [new information D]

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163. *See* Anuj K. Shah & Daniel M. Oppenheimer, *The Path of Least Resistance: Using Easy-to-Access Information*, 18 CURRENT DIRECTIONS PSYCHOL. SCI. 232, 233 (2009) (discussing relationship between perception and fluency). "Fluency often increases when information is presented in familiar ways." *See id.*; *see also* Norbert Schwarz, *Metacognitive Experiences in Consumer Judgment and Decision Making*, 14 J. CONSUMER PSYCHOL. 332, 339 (2004) ("[f]luency-based familiarity inferences have important implications for judgments of truth").

164. *See* Lucille A. Jewel, *Old-School Rhetoric and New-School Cognitive Science: The Enduring Power of Logocentric Categories*, 13 LEGAL COMM. & RHETORIC: JALWD 39, 68 (2016) (explaining utility of categories and syllogisms).

165. *See id.* at 65 (acknowledging fluency through repetition).

166. *See id.* at 68.

167. *See* Dale W. Holloway, *Semantic Grammars: How They Can Help Us Teach Writing*, 32 C. COMPOSITION & COMM. 205, 207 (1981) (discussing "given-new" sentence structure).

168. *See id.* at 207-08 (noting "*old* information regularly appears at . . . *beginning* of . . . sentence, and *new* information at . . . *end*").

169. *See* Robert C. Weissberg, *Given and New: Paragraph Development Models from Scientific English*, 18 TESOL Q. 485, 488 (1984) (describing studies showing test subjects understood and recalled content faster when content followed "given-new" structure).

170. *See* Holloway, *supra* note 167, at 208.

Alternatively, the sentences in a paragraph can be structured such that each sentence adds to the given information in the first sentence.[171]

Sentence 1: [given information A] [new information B]

Sentence 2: [given information A] [new information C]

Sentence 3: [given information A] [new information D]

Aside from processing the words on the page, readers of a legal analysis or mathematical analysis will have an easier time following the argument if it uses an inferential rule known as *modus ponens*. *Modus ponens* is a rule of inference that applies when an argument takes the following form: "If p, then q; p, therefore, q."[172] Using the terms of a syllogism, the major premise is, "If p, then q."[173] The minor premise is "p."[174] And the conclusion is "q."[175] More concretely, consider the following two statements:

1. If the temperature is below 32 degrees Fahrenheit, then the water will freeze.
2. The temperature is below 32 degrees Fahrenheit.

Applying *modus ponens*, one can infer that the water will freeze.

Based on studies involving problem solving using the *modus ponens* rule of inference, "[f]or *modus ponens*, there is evidence that people: (a) perform as well—that is, make inferences in accordance with the rule—on unfamiliar as on familiar material; [and] (b) perform as well on abstract as on concrete material."[176] The freezing water example above is both concrete and familiar—we know and understand the freezing point of water. Subjects in the studies, however, were able to apply the logic of *modus ponens* even on material that was abstract or less familiar.[177] Thus, reasoning that relies on *modus ponens* should be easy for the reader to follow even when the reader is not very familiar with the underlying material.

171. *See id.* at 209.

172. *See* Barbara A. Kalinowski, *Logic* Ab Initio*: A Functional Approach to Improve Law Students' Critical Thinking Skills*, 22 LEGAL WRITING: J. LEGAL WRITING INST. 109, 130 (2018) (explaining conditional syllogisms).

173. *See id.* at 128 (describing parts of syllogism).

174. *See id.*

175. *See id.*

176. *See* Edward E. Smith et al., *The Case for Rules in Reasoning*, 16 COGNITIVE SCI. 1, 30 (1992).

177. *See id.* at 11, 15. Examples of abstract and less familiar rules include the following: "If the letter is L, then the number is 5" and "[i]f she meets her friend, then she will go to a play." *Id.* Although the friend going to a play example is not nonsensical, it is less "familiar" because meeting with a friend does not necessarily always lead to going to a play. Many people meet friends and then go for coffee, to lunch, to a movie, or to do any of hundreds of other activities.

V. WHAT CAN LAWYERS LEARN FROM MATHEMATICIANS?

In light of the similarities between written legal analysis and written mathematical analysis, are there further insights to be gleaned from mathematics, especially in areas where the two forms of writing differ?

One difference between the two forms lies in how and where the facts are incorporated. In IRAC, the facts of the case at hand do not appear until the A section, when the facts are analyzed in light of the rules and the precedent.[178] Although many forms of legal writing—such as memos and briefs—call for a fact section before the legal analysis (i.e., before any IRAC), an IRAC-organized analysis does not incorporate facts until A.[179] The IRAC in Figure 3 above is relying on facts that the reader likely already knows from seeing them earlier in the document.[180] Within the IRAC, however, those facts do not appear until the application section.[181]

With mathematical proof, on the other hand, the givens are announced at the beginning of the proof, serving as an immediate starting place for the analysis.[182] Specifically, a proof typically starts with either the givens and the statement to be proven,[183] or with a statement of the theorem, which includes both the givens and the statement to be proven.[184] In terms more familiar to legal writers, the proof starts with the issue and the facts. The facts are then immediately used and analyzed. In Figure 2 above, the given is that x and y are even integers.[185] The proof immediately uses that fact to reason that, based on the definition of even integers, there must be another integer n such that $x = 2n$ and an integer m such that $y = 2m$.[186]

Legal writers should consider whether it is appropriate to include a discussion of the facts before the A of IRAC. Following the mathematical model, the facts could come either right before or after I, the statement of the issue. Indeed, as Professor Diane Kraft discovered when she analyzed the IRAC form of federal appellate briefs, many legal writers already do this.[187]

Another difference between legal analysis and mathematical analysis is in the interplay between the rules and the application of the rules. In IRAC, all of the relevant rules are discussed in the R section before the writer moves on to A to

178. *See supra* Section II.A.2.

179. *See, e.g.*, CALLEROS, *supra* note 42, at 152-53, 339, 369; SHAPO ET AL., *supra* note 44, at 339.

180. *See supra* Figure 3. The analysis set forth in Figure 3 relies on the facts that Ms. Garcia and Mr. Jordan promised each other that they would be "like husband and wife" and that the seventeen witnesses at trial all testified that they considered Ms. Garcia and Mr. Jordan to be husband and wife.

181. *See supra* Figure 3.

182. *See supra* Section II.A.1 (explaining purpose of givens in mathematical proof).

183. *See supra* Figure 1.

184. *See supra* Figure 2.

185. *See supra* note 39 and accompanying text (defining "even number").

186. *See id.*

187. *See* Diane B. Kraft, *CREAC in the Real World*, 63 CLEV. ST. L. REV. 567, 585-86 (2015) (describing instances when facts get incorporated into legal analysis).

apply those rules.[188] In a mathematical proof, in contrast, the writer moves back and forth between the rules and the application throughout the document. Each step of the mathematical proof includes both a rule and an application of the rule. The analysis continues step-by-step until the writer reaches the desired conclusion.[189] In IRAC terms, a proof would look more like IRA-RA-RA-RA-RA-C rather than the standard IRAC.

Some of the difference here between mathematical proof and IRAC is due to the fact that the rules in law are more complex and often have multiple parts, elements, or factors. It may not make sense to separate those pieces because they all work together. Nonetheless, legal writers should consider whether and when a more step-wise analysis could be appropriate. Furthermore, even when the complexity of the rules does not allow this approach, legal writers should use the "given–new" sentence structure within their larger R's and A's.[190] As Professors Catherine Cameron and Lance Long explain in their book, *The Science Behind the Art of Legal Writing*, "a legal writer can be guided by what we know about good organization on the paragraph level—beginning the paragraph with a topic sentence . . . and organizing sentences in a manner that takes advantage of the given-new structure will be the most effective organization for a reader."[191]

Legal writers should consider IRAC to be a flexible organizational scheme rather than a rigid formula. To the extent that, with any given argument, it is helpful to lay out the facts earlier in IRAC or to move back and forth between R and A multiple times, legal writers should do so.

VI. CONCLUSION

Law and mathematics, though seemingly very different, are built on the same core of reasoning and analysis. With some understanding of the methods of mathematical proof, lawyers can harness their inner (possibly dormant) mathematicians to develop solid, convincing analyses.

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188. *See* COUGHLIN ET AL., *supra* note 52, at 83-85 (noting "application . . . include[s] [no] . . . new law").

189. *See supra* Figures 1, 2 (demonstrating proofs and two-column proofs).

190. *See supra* notes 165-170 and accompanying text (describing importance of "given-new" information structure to fluency).

191. CATHERINE J. CAMERON & LANCE N. LONG, THE SCIENCE BEHIND THE ART OF LEGAL WRITING 91 (2015).

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